

Exercice

Résoudre dans $\mathbb{C}$ les équations suivantes :

1. $3iz + 2 = 5z$

2. $(2 + 5i)z + 1 + i = (1 + 2i)z$

3. $\frac{z}{2+i} + 1 = \frac{z}{1-i} + i$

Correction :

1. $(-5+3i)z = -2$

$$z = \frac{-2}{-5+3i} = \frac{-2(-5-3i)}{(-5+3i)(-5-3i)} = \frac{10+6i}{25+9} = \frac{5}{17} + \frac{3}{17}i$$

$$S = \left\{ \frac{5}{17} + \frac{3}{17}i \right\}$$

2. $(2+5i)z + 1 + i = (1+2i)z$

$$[(2+5i) - (1+2i)]z = -1 - i$$

$$(1+3i)z = -1 - i$$

$$z = \frac{-1-i}{1+3i} = \frac{(-1-i)(1-3i)}{(1+3i)(1-3i)} = \frac{-1+3i-i-3}{1+9} = \frac{-4+2i}{10} = \frac{-2}{5} + \frac{1}{5}i$$

$$S = \left\{ -\frac{2}{5} + \frac{1}{5}i \right\}$$

$$3. \frac{z}{2+i} + 1 = \frac{z}{1-i} + i$$

$$\left(\frac{1}{2+i} - \frac{1}{1-i} \right) z = -1 + i$$

$$\left(\frac{2-i}{(2+i)(2-i)} - \frac{1+i}{(1-i)(1+i)} \right) z = -1 + i$$

$$\left(\frac{2-i}{5} - \frac{1+i}{2} \right) z = -1 + i$$

$$\left(\frac{2(2-i) - 5(1+i)}{10} \right) z = -1 + i$$

$$\left(\frac{-1-7i}{10} \right) z = -1 + i$$

$$z = \frac{10(-1+i)}{-1-7i} = \frac{(-10+10i)(-1+7i)}{(-1-7i)(-1+7i)} = \frac{10-70i-10i-70}{50} = \frac{-60-80i}{50} = \frac{-6}{5} - \frac{8}{5}i$$

$$S = \left\{ -\frac{6}{5} - \frac{8}{5}i \right\}$$